BRIEF COMMUNICATIONS

# PARTICLE MOTION IN THE SOUND FIELD OF A STANDING WAVE

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The problem of the motion of a particle in a standing sound wave is solved with allowance for the nonuniformity of particle motion under the action of a variable force. A new equation of particle motion is obtained for this case.

The relation

$$\operatorname{tg}(KX) = \operatorname{tg}(KX_0)\exp B_1 t, \tag{1}$$

where

$$B_1 = \frac{5}{9}\,\frac{K^2R^2E}{\eta}, \tag{2}$$

was obtained [1] to describe the motion of a particle acted upon by the radiation force of the sound field of a standing wave

$$F = \frac{5}{3}\,K\pi R^3E\sin(2KX), \tag{3}$$

but without allowance for the second derivative with respect to time.

According to [2], in a plane standing wave the radiation force acting on a small spherical particle is given by

$$F = 4\pi KER^3\psi\sin(2KX), \tag{4}$$

where

$$\psi = \frac{\rho_0 + \frac{2}{3}(\rho_0 - \rho)}{2\rho_0 + \rho} - \frac{C^2\rho}{C_0^2\rho_0}.$$

An analysis of the radiation force leads to the conclusion that when $\rho > \rho_0$, the particles are bunched at the velocity antinode and at the velocity nodes $\rho < \rho_0$ [3].

According to [4], in computing the viscosity, we assume the small spherical liquid particles to be solid. Under these conditions the friction force expressed in terms of the Stokes law is

$$F = 6\pi\eta R\dot{X}. \tag{5}$$

The differential equation of particle motion with allowance for the second derivative is

$$\ddot{\xi} + \mu\dot{\xi} - b\sin\xi = 0, \tag{6}$$

where we have introduced the notation $\mu = (9/2)(\eta/R^2\rho)$, $b = (6/\rho)K^2E\psi$; these quantities are constant for a given medium, given particles, and given acoustic energy density; $\xi = 2KX$ is a new variable. With the initial conditions

$$t = 0,\ \dot{\xi} = 0,\ \xi = \xi_0 \tag{7}$$

this equation has the following solution:

$$t = \frac{1+\mu^2}{b}\int\limits_{\xi_0}^{\xi}\Big\{\mu\sin\xi - \cos\xi - \frac{b\mu}{1+\mu^2}(\mu\sin\xi_0 - \cos\xi_0)\exp[-\mu(\xi - \xi_0)]\Big\}^{-1}d\xi. \tag{8}$$

If $R \ll 1$, then $\mu \gg 1$, and it can be shown that the last term in the denominator of the integrand is much smaller than the first two terms over the entire interval of particle motion. Taking this into account, we obtain

$$t = \frac{\sqrt{1+\mu^2}}{b}\left[\ln\left|\operatorname{tg}\frac{\xi+\theta}{2}\right| - \ln\left|\operatorname{tg}\frac{\xi_0+\theta}{2}\right|\right]. \tag{9}$$

It can be shown [5] that $\theta$ and $\mu$ are related by

$$\cos\theta = \frac{\mu}{\sqrt{1+\mu^2}},\quad \sin\theta = -\frac{1}{\sqrt{1+\mu^2}}. \tag{10}$$

We rewrite Eq. (9) as

$$\operatorname{tg}\left(KX + \frac{\theta}{2}\right) = \operatorname{tg}\left(KX_0 + \frac{\theta}{2}\right)\exp Bt, \tag{11}$$

where

$$B = \frac{12K^2R^2E}{\sqrt{4\rho^2R^4 + 81\eta^2}}. \tag{12}$$

Thus, we conclude that the velocity of small spherical particles in a plane standing sound wave depends on the position of the latter (8) relative to the node or antinode.

Consideration of the nonuniform motion of the particle shows that the exponent in Eq. (11) is expressed by relation (12) rather than by relation (2).

From Eq. (11) it follows that the particles are bunched in the region of the node or antinode (depending on the relation between $\rho_0$ and $\rho$) our the interval $\theta/2$, where $\theta$ is determined from (10), while the exponent is a function of the particle density and of the radius in a manner different from that given in [1] (2). The condition $R \ll 1$ is quite accurately satisfied for all emulsions and suspensions, since the radius of the particle in highly dispersed emulsions and suspensions lies in the range from 1 $\mu$ to several tens of microns, i.e., on the order of $10^{-6}$ m.

## NOTATION

F is the mean force; E is the mean sound energy density; K is the wave number; R is the particle radius;

X is the coordinate normal to the wave front; $\rho_0$ is the density of the medium; $\rho$ is the density of the particle; $C_0$ is the speed of sound in the medium; C is the speed of sound in the particle; $\dot{X}$ is the particle velocity; $\ddot{X}$ is the particle acceleration; t is the time; and $\eta$ is the liquid viscosity.

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# RAYLEIGH EQUATION FOR THE GROWTH OF A GAS BUBBLE UNDER CONDITIONS OF FINITE LIQUID VOLUME

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An equation is presented for the growth of a stationary spherical gas bubble in a finite spherical liquid volume, for which the familiar Rayleigh equation is the zero-order approximation. The results of computer solutions of the derived equation and the Rayleigh equation are compared.

In [1] Rayleigh derived an equation for the growth of a stationary spherical gas bubble in an infinite volume of liquid. Thus, from the equations of hydrodynamics and continuity in a spherical coordinate system

$$\rho\left(\frac{\partial v}{\partial t}+v\,\frac{\partial v}{\partial r}\right)=-\frac{\partial P}{\partial r},$$

$$\frac{\partial v}{\partial r}+\frac{2v}{r}=0$$

with boundary condition $v|_{r=R}=\dot{R}(t)$, where, as usual, $\dot{x}\equiv dx/dt$, one obtains

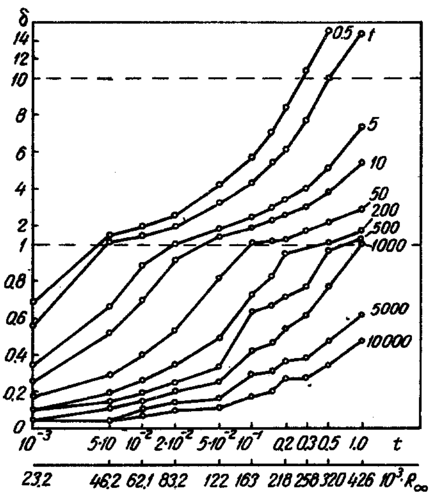
$$\frac{\ddot{R}R^2+2R\dot{R}^2}{r^2}-\frac{2R^4\dot{R}^2}{r^5}=-\frac{1}{\rho}\,\frac{\partial P}{\partial r}. \qquad (1)$$

Then Rayleigh, integrating (1) from $r = R$ to $r = \infty$, obtained his familiar equation

$$R\ddot{R}+\frac{3}{2}\dot{R}^2=\frac{1}{\rho}\,(P_R-P_\infty). \qquad (2)$$

Equation (2) has been used by many authors (for example, [2–5]) in connection with problems involving the growth of a bubble in a liquid. However, the finiteness of the liquid volume has not been taken into account.

To estimate the effect of this factor on the bubble growth and to compare with the Rayleigh solution, we




Deviation of the solutions for $R_Q$ in Eqs. (4) with Q = var from the solution $R_\infty$ of the Rayleigh equation (for $Q = \infty$). Plotted along the axis of abscissas, to a logarithmic scale, are the time t (sec) and the corresponding radius $R_\infty$ ($\cdot 10^3$, cm); along the ordinate axis, to a variable scale, we have the deviation in % of $R_\infty$ from $R_Q$ for the corresponding t. $\delta = (R_Q - R_\infty)\cdot 100/R_Q$.

formulate the simple following problem: at time $t = 0$ a spherical gas bubble of radius $R_0$ is formed and begins to grow at the center of a spherical volume Q of liquid with infinite permeability at the boundary, the